Patterns and Algebra – AP Book 7, Part 1: Unit 2

AP Book PA7-1

page 22

1. a) +3; 10, 13, 16
   b) +6; 20, 26, 32
   c) +5; 16, 21, 26
   d) +4; 15, 19, 23
2. a) –3; 8, 5, 2
   b) –6; 13, 7, 1
   c) –5; 36, 31, 26
   d) –4; 21, 17, 13
3. a) +1 +3 +5 +7 +9; 26
   b) –5 –4 –3 –2 –1; 5
   c) +1 +2 +4 +8 +16; 32
   d) –20 –15 –10 –5 0; 7
   e) +0 +1 +1 +2 +3 +5 +8 +13 +21 +34 +55; 55, 89, 144
4. Each term is the sum of the two previous terms.
5. *Gaps between gaps:*
   +4 +5 +6 +7 +8 +9
   Gaps:
   +9 +14 +20 +27 +35 +44
   Pattern:
   78, 113, 157

AP Book PA7-2

page 23

1. a) +4 –2 +7 –4
   b) +4 –1 +4 –5
   c) +4 +3 +10 +6
   d) +4 –1 –6 +9
   e) +5 +8 –6 –3
   f) +3 –7 +8 –5
2. a) B, A
   b) A, B
   c) A, B
   d) A, B
   e) C, B, A
   f) C, A, D, B
3. Answers will vary.
4. a) ×2; 8, 16

b) ×2; 40, 80
c) ×3; 54, 162
d) ×10; 7 000, 70 000
5. a) ÷2; 50, 25
   b) ÷2; 12, 6
   c) ÷5; 4, $\frac{4}{5}$
   d) ÷4; 25, $\frac{25}{4}$
6. a) Start at 3 and multiply by 2.
   b) Start at 4 and add 3.
   c) Start at 28 and subtract 3.
   d) Start at 27 and divide by 3.
   e) Start at 2 and multiply by 5.
   f) Start at 32 and divide by 2.
   g) Start at 30 000 and divide by 10.
   h) Start at 10 and multiply by 20.
7. a) Increasing
   b) Repeating
   c) Decreasing
   d) Increasing
   e) Repeating
   f) Decreasing
8. b) 0, 5 then repeat
   c) 11, 9, 6 then repeat
   d) M, M, N then repeat

AP Book PA7-3

page 25

1. a) *Gap:* +4; Start at 3 and add 4.
   b) *Gap:* +4; Start at 2 and add 4.
   c) *Gap:* +2; Start at 2 and add 2.
   d) *Gap:* +5; Start at 1 and add 5.
   e) *Gap:* +4; Start at 5 and add 4.
   f) *Gap:* +6; Start at 12 and add 6.
2. a) *Gap:* +5; 17, 22, 27

b) *Gap:* +3; 13, 16, 19
c) *Gap:* +5; 18, 23, 28
3. a) Yes (14)
   b) No (16)
   c) Yes (13)
4. a)

Figure	# of Squares
1	4
2	6
3	8
4	10
5	*12*

   b)

Figure	# of Squares
1	2
2	6
3	10
4	14
5	*18*

5. a) 5 cm
   b) 27 cm
   c) 6 weeks
6. a) 4 L
   b) 8 L
   c) 6 hours
7. 39 cm
8. $24
9. a)

Figure	# of TP
1	4
2	7
3	10
4	13
5	*16*

   b)

Figure	# of TP
1	5
2	9
3	13
4	17
5	*21*

10. a) 36
    b) 36
    c) 8 rhombuses and 16 triangles
    d) 24 triangles

11. Edith's
12. 5 weeks

AP Book PA7-4

page 28

1. a)

Figure	# Shaded
1	1
2	3
3	6
4	10
5	*15*

   b)

Figure	# Shaded
1	1
2	3
3	9
4	27
5	*81*

2. a) 0 1 1 2 3 5 8 13 21; 34, 55
   b)

5	8	13	21	*34*	*55*
O	E	O	O	*E*	O

   c) O, O, E then repeat
   d) It is odd since 38 is not a multiple of 3.
   e) They are equal.
   f) They are equal.
3. a) Each number is equal to the sum of the two above.
   b) 1 5 10 10 5 1
      1 6 15 20 15 6 1
   c) 28
   d) Answers will vary.

INVESTIGATION

A. a) 0 lines
   b) 1 line
   c) 3 lines
   d) 6 lines
B. a) 0
   b) +1, 1
   c) +2, 3
   d) +3, 6
C. *+1 +2 +3 +4 +5*
   0 1 3 6 10 15

Patterns and Algebra – AP Book 7, Part 1: Unit 2 *(continued)*

D. Teacher to check.

E. a) 28 lines
    b) 45 lines

AP Book PA7-5
page 30

1. Answer will vary.
2. a) 3s
    b) 5n + 2
    c) 12 – 4r
    d) 7a – 3
    e) 4b – 3
    f) 5 + 6w
3. a) 3 × h
    b) 2 – 3 × g
    c) 3 × f + 4
    d) 5 + 7 × t
    e) 7 × a – 4
    f) 3 × x + 4 × y
4. a) 3 × 2
    b) 3 × 5
    c) 3 × 6
5. a) 5 × h or 5h
    b) 5 × t or 5t
    c) 5 × x or 5x
6. a) 60 × 2
    b) 80 × 3
    c) 70 × 5
7. a) 70h
    b) 70t
    c) 70z
8. a) 2 × 5 + 9
    b) 3 × 6 + 4
    c) 7 × 4 + 5
    d) 4h + 5
    e) 3t + 8
    f) 5w + 6
9. a) 7h + 15
    b) 15h
    c) 15h + 7
10. a) Umbrellas; 2n
    b) Copy; 0.79n
    c) Passenger; 5n + 10
    d) Passenger; 7n + 20

11. a) n + 7
    b) n + 5
    c) n + 12
    d) n + 31
12. a) 5 + 7 = 12
    b) 5 + 5 = 10
    c) 5 + 12 = 17
    d) 5 + 31 = 36
13. No: John simply replaced the variable with the number of pairs, rather than multiplying.
14. a) 10
    b) 6
    c) 8
    d) 9
    e) 6
    f) 15
15. a) 11
    b) 11
    c) 4
    d) 7
16. a) 18
    b) 12
    c) 21
    d) 27
17. a) 17
    b) 15
    c) 18
    d) 33
    e) 1
    f) 17
18. a) 13
    b) 13
    c) 13
19. They are equal because they are exactly the same equations but with a different variable.
20. a) 19
    b) 19
21. They are equal because they're the same except the 2x and the 7 are switched around (and adding is commutative).

22. *Circle:*
    7 + 4n   4m + 7
    7 + 4w   7 + 4 × p
    n × 4 + 7
23. a) $23.00
    b) $30.00
    c) $20.00
24. a) 8h + 5
    b) 7 hours
        (8h + 5 = 61)

AP Book PA7-6
page 34

1. a) 3 4 5 6 7 8 9
        10 11 12 13
    b) i)   7
        ii)   1
        iii) 9
        iv)   4
        v)   0
2. a) 3n:
        6 9 12 15 18 21
        24 27 30 33 36
        3n – 5:
        1 4 7 10 13 16
        19 22 25 28 31
    b) i)   7
        ii)   10
        iii) 5
        iv)   12
3. a) 17; less than greater than
    b) 31; greater than less than
    c) 13; equal to equal to
    d) 23; less than greater than
4. a) 2
    b) 3
    c) 4
    d) 6
    e) 3
    f) 3
    g) 3
    h) 4
5. a) 7(8) + 11
        = 56 + 11 = 67

b) t = 8
c) x = 8; same equation, just the variable and order are different.
d) x = 8; same equation, just the order is different.

6. *Circle:*
    3 + 8x = 51
    8t + 3 = 51
    51 = 3 + 8x
    51 = 8w + 3
    r × 8 + 3 = 51
    51 = 3 + 8t
    3 + 8r = 51
    8z + 3 = 51
7. a) 3
    b) 2
    c) 4
    d) 3
    e) 3
    f) 7
    g) 8
    h) 5
    i) 3
    j) 4
    k) 5
    l) 6
    m) 3
    n) 3
    o) 5
    p) 3
8. n)
BONUS   x = 3; y = 2

AP Book PA7-7
page 36

1   a) x + 2
    b) 3x
    c) x + 3
2. a) x + 2 = 7
    b) 2x + 4 = 10
    c) 3x = 15
3. a) 3x + 1 = 10; x = 3
    b) 2x + 3 = 13; x = 5

c) $5x + 2 = 17$; $x = 3$

d) $3x + 2 = 11$; $x = 3$

e) $4x + 2 = 14$; $x = 3$

f) $3x + 7 = 31$; $x = 8$

4. a) $x = 5$
Models will vary.

b) $x = 4$

c) $x = 3$

d) $x = 2$

e) $x = 5$

f) $x = 3$

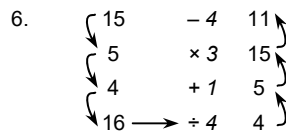5. No, because you can't draw the "subtracted" apples.

AP Book PA7-8
page 37

1. a) 4
b) 3
c) 2
d) 5
e) 4
f) 6
g) 5
h) 3

2. a) –
b) ÷
c) ×
d) +
e) ×
f) +
g) ÷
h) –

3. a) – 3
b) × 4
c) ÷ 2
d) + 4
e) ÷ 3
f) – 2
g) ÷ 2
h) – 3
i) ÷ 3
j) × 2
k) ÷ 2
l) + 2

4. a) Subtract 4
b) Divide by 3
c) Multiply by 2
d) Add 7

5.
10	→	3
20	→	10
15	→	20
5	→	15

Yes

6.
15	– 4	11
5	× 3	15
4	+ 1	5
16	÷ 4	4
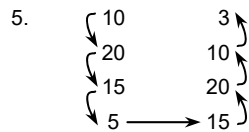
Yes

7. a) $7x$
b) $x + 4$
c) $x - 5$
d) $5 - x$
e) $x \div 10$
f) $9 \div x$
g) $8x$
h) $x + 9$
BONUS $y + x$

8. a) Multiply by 2
b) Multiply by 3
c) Add 4
d) Subtract 5
e) Divide by 3
f) Divide into 6
g) Subtract from 4
BONUS Add to itself

9. a) – 3
b) ÷ 3
c) ÷ 5
d) + 5
e) – 7
f) + 14
g) × 5
h) ÷ 7
i) – 8
j) – 4
k) ÷ 6
l) – 4

10. *Circle:*
$7m \div 7$
$m \div 7 \times 7$
$7 + m - 7$

11. a) 4
b) 15
c) 5
d) 6
e) 3
f) 18
g) 3
h) 17
i) 9
j) 6
k) 19
l) 9
m) 6
n) 27

AP Book PA7-9
page 39

1. b)

x	$8x + 5 = 37$
$8x$	$8x + 5 - 5$ $= 37 - 5$
$8x + 5$	$8x = 32$
$8x + 5 = 37$	$8x \div 8$ $= 32 \div 8$
	$x = 4$

Check: 4; 32; 37; Yes

c)

x	$4x - 3 = 37$
$4x$	$4x - 3 + 3$ $= 37 + 3$
$4x - 3$	$4x = 40$
$4x - 3 = 37$	$4x \div 4$ $= 40 \div 4$
	$x = 10$

Check: 10; 40; 37; Yes

2. a) $8x + 3 = 27$
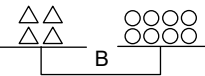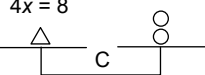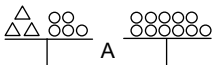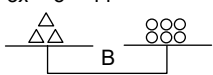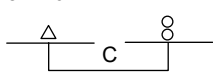$8x + 3 - 3 = 27 - 3$
$8x = 24$
$8x \div 8 = 24 \div 8$
$x = 3$

b) $4h - 3 = 37$
$4h - 3 + 3 = 37 + 3$
$4h = 40$
$4h \div 4 = 40 \div 4$
$h = 10$

c) $3s - 4 = 29$
$3s - 4 + 4 = 29 + 4$
$3s = 33$
$s = 11$

d) $2t + 3 = 11$
$2t + 3 - 3 = 11 - 3$
$2t = 8$
$2t \div 2 = 8 \div 2$
$t = 4$

e) $2m - 7 = 13$
$2m - 7 + 7 = 13 + 7$
$2m = 20$
$2m \div 2 = 20 \div 2$
$m = 10$

f) $5a + 2 = 47$
$5a + 2 - 2 = 47 - 2$
$5a = 45$
$5a \div 5 = 45 \div 5$
$a = 9$

g) $4z + 3 = 19$
$4z + 3 - 3 = 19 - 3$
$4z = 16$
$4z \div 4 = 16 \div 4$
$z = 4$

h) $7w - 2 = 26$
$7w - 2 + 2 = 26 + 2$
$7w = 28$
$7w \div 7 = 28 \div 7$
$x = 4$

i) $8x + 3 = 3$
$8x + 3 - 3 = 3 - 3$
$8x = 0$
$8x \div 8 = 0 \div 8$
$x = 0$

j) $9r - 3 = 6$
$9r - 3 + 3 = 6 + 3$
$9r = 9$
$9r \div 9 = 9 \div 9$
$r = 1$

3. a) 3*h*
    b) $12
    c) 5 hrs
4. a) 9*h* + 36
    b) $63
    c) 6 hrs
5. a) in b)
    b) in c)
6. a) A: 3*n* + 100
        B: 5*n* + 120
        C: 7*n* + 70
    b) 8 people
    c) A: $124
        B: $160
   d) No: company A had the best price.

AP Book PA7-10
page 41

1. a) 2*x* + 3 = 9
    b) 3*x* + 1 = 10
    c) 4*x* + 1 = 9
2. a) 3*x* + 2 = 8; 3*x* = 6
    b) 2*x* + 1 = 9; 2*x* = 8
3. a) 3*x* = 6; *x* = 2
    b) 2*x* = 6; *x* = 3
    c) 4*x* = 8; *x* = 2
    d) 3*x* = 12; *x* = 4
4. a) 3*x* + 2 = 8; 3*x* = 6; *x* = 2
    b) 4*x* + 1 = 9

        4*x* = 8

        x = 2
5.

        3*x* + 5 = 11

        3*x* = 6

        x = 2

AP Book PA7-11
page 42

1. a) 6*x* = 3*x* + 3*x*
    b) 6*x* = 2*x* + 4*x*
    c) 6*x* = *x* + 5*x*
2. a) 3*x* + *x* = 4*x*
    b) 5*x* + 2*x* = 7*x*
    c) 7*x* + *x* = 8*x*
    d) 4*x* + 2*x* + 3*x* = 9*x*
3. a) *x* = 3
    b) *x* = 2
    c) *x* = 4
    d) *x* = 4
4. a) 0
    b) 0
    c) 0
    d) 0
    e) 3
    f) 7
    g) 5
    h) *x*
5. a) 4
    b) 5
    c) 7
    d) 8
    e) 7
    f) 9
    g) 4
    h) 4
    i) 7
    j) 12
    k) *x*
    l) 2*x*
    m) 2*x*
6. a) 3*x*
    b) 3*x*
    c) 6*x*
7. a) 2*x*
    b) 4*x*
    c) 5*x*
    d) 10*x*
8. a) *x* = 5
    b) *x* = 5
    c) *x* = 5

AP Book PA7-12
page 43

1. Circle the (equations):
        n + 6 = 7
        7 + 3*n* = 5 + 2*n*
        a + *b* = *b* + *a*
        8 − 3*n* = 5
        a × *b* = *b* × *a*
    Underline the expressions:
        5*n* − 3
        7 + 3*n*
        a + *b*
        4 + 6*n* − 5*m*
2. An equation uses an equals sign to equate two expressions.
3. They both contain one or more expressions.
4. Answers will vary, example: 5*n* + 3 = 8
5. 5*n* is changing because *n* is the variable.
6. a) 2
    b) 1
    c) 4
    d) 7
7. a) 3
    b) 5
    c) 4
    d) 1
8. a) 7
    b) 0
    c) 9
    d) 5
9. a) 3*h* + 10
    b) 3
    c) 10
10. a) 50*h*
    b) 0
    c) 50
    d) The number of hours the car travels.
11. a) 13 apples in total:
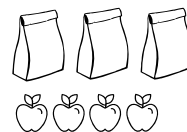
    b) coefficient
    c) variable
    d) constant term
    e) 13
12. coefficient →
        the hourly rate
    constant term →
        the flat fee
    variable →
        the # of hours rented

AP Book PA7-13
page 45

1. a) 2
    b) 2
    c) 3
    d) 3
    e) 10
2. a) *x* = 4
    b) *x* = 5
    c) *x* = 3
    d) *x* = 7
    e) *x* = 7
    f) *x* = 4
    g) *x* = 3
    h) *x* = 7
    i) *x* = 6
    j) *x* = 8
3. a) *x* = 12
    b) *x* = 10
    c) *x* = 9
    d) *x* = 14
    e) *x* = 36
4. a) *x* = 6
    b) *x* = 4
    c) *x* = 20
    d) *x* = 5
    e) *x* = 11
    f) *x* = 12
    g) *x* = 3
    h) *x* = 6
    i) *x* = 5
    j) *x* = 12

AP Book PA7-14
page 46

1. a) 2 more than a number → $x + 2$

    a number divided by 3 → $x \div 3$

    2 less than a number → $x - 2$

    the product of a number and 4 → $4x$

    a number decreased by 3 → $x - 3$

    b) 2 divided into a number → $x \div 2$

    a number reduced by 4 → $x - 4$

    a number times 3 → $3x$

    twice as many as a number → $2x$

    a number increased by 3 → $x + 3$

2. a) $x + 4$
    b) $x - 10$
    c) $7x$
    d) $x \div 8$
    e) $x - 2$
    f) $x + 7$
    g) $x + 9$
    h) $x - 4$
    i) $3x$
    j) $5x$
    k) $x \div 6$
    l) $2x$

3. a) $x + 4 = 18$; $x = 14$
    b) $x - 5 = 12$; $x = 17$
    c) $5x = 30$; $x = 6$
    d) $6x = 42$; $x = 7$
    e) $x \div 6 = 4$; $x = 24$
    f) $5x = 40$; $x = 8$
    g) $2x + 5 = 35$; $x = 15$
    h) $3x - 4 = 17$; $x = 7$
    i) $x \div 3 + 2 = 8$; $x = 18$
    j) $2x = 10 + 4$; $x = 7$
    k) $3x = 28 - 4$; $x = 8$

BONUS

$x \div 2 = 3 + 5$; $x = 16$

4. a) 5
    b) 8
    c) 39
    d) $x + 2$

5. a) 6
    b) 9
    c) 22
    d) 37
    e) $x + 2$, $x + 4$
    f) $x + 2$, $x + 4$

6. a)

Numbers	Sum
4, 5	9
5, 6	11
6, 7	13
7, 8	15
8, 9	17
9, 10	19
10, 11	21
11, 12	23
12, 13	25
13, 14	27
14, 15	29
15, 16	31
16, 17	33
17, 18	35

    b) i) x
           $x + 1$
        ii) $x + x + 1$
             $= 2x + 1 = 35$
        iii) $2x = 34$ so $x = 17$; the two numbers are 17 and 18

7. a) $x + x + 2 = 32$
        $2x = 30$
        $x = 15$
        ∴ 15 and 17
    b) $x + x - 2 = 32$
        $2x = 34$
        $x = 17$
        ∴ 17 and 15
    c) Yes

8. a)

Numbers	Sum
1, 3	4
3, 5	8
5, 7	12
7, 9	16
9, 11	20
11, 13	24
13, 15	28
15, 17	32

    b) Yes
    c) Answers may vary, but algebra is generally faster and easier.

9. $x + x + 2 + x + 4 = 42$
    $3x = 36$
    $x = 12$
    ∴ 12, 14 and 16

10. a) i) $A = 10$
        ii) $A = 63$
        iii) $A = 150$
    b) $w = A \div l$
        $w = 56 \div 8$
        $w = 7$

11. a) i) $P = 3x$
        ii) $P = 4x$
        iii) $P = 2x + 10$
        iv) $P = 2x + 12$
    b) i) $x = 8$
        ii) $x = 6$
        iii) $x = 7$
        iv) $x = 6$

12. $x + x + 5 = 31$
    $2x = 26$
    $x = 13$
    ∴ 13 and 18 years old

13. $x + 3x = 48$
    $4x = 48$
    $x = 12$
    ∴ 12 years old

BONUS

$x + x - 3 + 3x + 3x + 4 = 89$

$8x + 1 = 89$

$8x = 88$

$x = 11$

∴ Bilal is 11 now and his mother is 33.

$33 - 11 = 22$

So Bilal's mother was 22 when he was born.

AP Book PA7-15
page 49

1. b) $m + 3 - 3 = m$
    c) $7 + m - m = 7$
    d) $m \div 2 \times 2 = m$
    e) $m - 5 + 5 = m$
    f) $9 - m + m = 9$

2. a) It is not true for $a = 0$.
    b) Answers will vary.

3. a) 9
    b) 15
    c) 36
    d) 4
    e) 36
    f) $\frac{1}{4}$

4. Circle c) and e).
    a) Yes; answer = 28
    b) Answer will vary.

5. $a \times b = b \times a$

6. b) $3 \times (5 + 3)$
        $= 3 \times 5 + 3 \times 3$
        $3 \times 8 = 15 + 9$
        $24 = 24$
    c) $3 \times (5 + 4)$
        $= 3 \times 5 + 3 \times 4$
        $3 \times 9 = 15 + 12$
        $27 = 27$
    d) $3 \times (5 + 5)$
        $= 3 \times 5 + 3 \times 5$
        $3 \times 10 = 15 + 15$
        $30 = 30$

e) $3 \times (5 + 6)$

$= 3 \times 5 + 3 \times 6$

$3 \times 11 = 15 + 18$

$33 = 33$

7. a) 3; 5; 4

b) 3; 5; 5

c) 3; 5; 6

8. a) 5; 2; 7

b) 3; 2; 5

c) 2; 6; 7

d) 4; 12; 83

9. a) $3 \times 1 + 3 \times 5$

b) $4 \times 2 + 4 \times 6$

c) $7 \times 0 + 7 \times 3$

d) $0 \times 3 + 0 \times 4$

e) $9 \times 2 + 9 \times 7$

f) $10 \times 1 + 10 \times 1$

10. a) 32; 32

b) 5; 5

c) 70; 70

The answers are
the same so:
$(a + b) \times c$
$= a \times c + b \times c$

NOTE: This is the
distributive law.

11. a) 5, 3, 6;
$(5 + 3) \times 6$
$= 5 \times 6 + 3 \times 6$

b) 8, 5, 3;
$(8 + 5) \times 3$
$= 8 \times 3 + 5 \times 3$

c) 7, 2, 9;
$(7 + 2) \times 9$
$= 7 \times 9 + 2 \times 9$

d) 3, 2, 0;
$(3 + 2) \times 0$
$= 3 \times 0 + 2 \times 0$