

# Patterns and Algebra – AP Book 7, Part 2: Unit 4

## AP Book PA7-16

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1. a)

s	$4 \times s = t$	t
1	$4 \times 1 = 4$	4
2	$4 \times 2 = 8$	8
3	$4 \times 3 = 12$	12

b)

s	$7 \times s = t$	t
1	$7 \times 1 = 7$	7
2	$7 \times 2 = 14$	14
3	$7 \times 3 = 21$	21

c)

s	$5 \times s = t$	t
2	$5 \times 2 = 10$	10
3	$5 \times 3 = 15$	15
4	$5 \times 4 = 20$	20

d)

s	$4 \times s = t$	t
5	$4 \times 5 = 20$	20
6	$4 \times 6 = 24$	24
7	$4 \times 7 = 28$	28

e)

s	$6 \times s = t$	t
1	$6 \times 1 = 6$	6
3	$6 \times 3 = 18$	18
5	$6 \times 5 = 30$	30

f)

s	$6 \times s = t$	t
9	$6 \times 9 = 54$	54
12	$6 \times 12 = 72$	72
15	$6 \times 15 = 90$	90

2. a)  $t = s \times 4$   
 b)  $t = s \times 5$   
 c)  $t = s \times 2$   
 d)  $t = s \times 6$

3. a)

s	r
1	4
2	8
3	12

$4 \times s = r$

b)

s	r
1	6
2	12
3	18

$6 \times s = r$

c)

s	t
1	8
2	16
3	24

$8 \times s = t$

d)

s	t
1	12
2	24
3	36

$12 \times s = t$

**BONUS**

s	t	r
1	4	4
2	8	8
3	12	12

$t = 4 \times s; r = 4 \times s$

$t = 1 \times r$

4. Wendy will need 28 rectangles.  
 5. No, since  $6 \times 7 = 42$ .  
 6. Teacher to check.  
 7. Teacher to check.

8. a)

$r + 6 = c$	c
$1 + 6 = 7$	7
$2 + 6 = 8$	8
$3 + 6 = 9$	9

b)

$r + 9 = c$	c
$1 + 9 = 10$	10
$2 + 9 = 11$	11
$3 + 9 = 12$	12

9. b)  $r + 7 = c$   
 c)  $r + 8 = c$   
 d)  $r + 5 = c$

10. a)

r	c
1	4
2	5
3	6

$r + 3 = c$

b)

r	c
1	2
2	3
3	4

$r + 1 = c$

11. There would be 13 chairs.

12. b)

Input	Output
4	0
5	1
6	2

c)

Input	Output
4	24
7	42
8	48

d)

Input	Output
28	7
16	4
40	10

e)

Input	Output
18	27
19	28
20	29

f)

Input	Output
4	32
6	48
9	72

g)

Input	Output
26	29
11	14
46	49

h)

Input	Output
15	10
19	14
23	18

i)

Input	Output
4	16
7	28
12	48

13. a)  $l \times 6 = O$   
 b)  $l \div 8 = O$   
 c)  $l - 3 = O$

- d)  $l + 3 = O$   
 e)  $l + 5 = O$   
 f)  $l \times 7 = O$

## AP Book PA7-17

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1. a) Multiply by 3;  
Divide by 3  
 b) Add 5;  
Subtract 5  
 c) Multiply by 2;  
Divide by 2  
 2. Multiply becomes divide  
(and vice versa); addition  
becomes subtraction (and  
vice versa)  
 3. a) (4,6), (5,7), (6,8)  
 b) (10,5), (8,4), (6,3),  
(4,2)  
 c) (5,10), (4,8), (3,6),  
(2,4)

## AP Book PA7-18

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1. a)

OP	1 <sup>st</sup>	2 <sup>nd</sup>
(2,1)	2	1
(4,3)	4	3
(6,5)	6	5

b)

OP	1 <sup>st</sup>	2 <sup>nd</sup>
(1,3)	1	3
(3,5)	3	5
(5,7)	5	7

c)

OP	1 <sup>st</sup>	2 <sup>nd</sup>
(2,4)	2	4
(4,5)	4	5
(6,6)	6	6

2. Teacher to check points  
marked on line segments.

a)

OP	1 <sup>st</sup>	2 <sup>nd</sup>
(0,1)	0	1
(1,3)	1	3
(2,5)	2	5
(3,7)	3	7

or

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b)

OP	1 <sup>st</sup>	2 <sup>nd</sup>
(0,5)	0	5
(2,6)	2	6
(4,7)	4	7

c)

OP	1 <sup>st</sup>	2 <sup>nd</sup>
(0,0)	0	0
(1,3)	1	3
(2,6)	2	6

3. Teacher to check graph.  
*Ordered Pairs:*  
 (3,1), (4,3), (5,5), (6,7)
4. Teacher to check.
5. Teacher to check.

## AP Book PA7-19

page 84

1. a) 7  
 b) 17  
 c) 11

2. a)

Term #	Term
1	1
2	3
3	5
4	7

b)

Term #	Term
1	2
2	4
3	6
4	8

c)

Term #	Term
1	4
2	7
3	10
4	13

3. b) (1,2), (2,4), (3,6), (4,8), (5,10)  
 c) (1,4), (2,7), (3,10), (4,13), (5,16)  
 d) (1,3), (2,7), (3,11), (4,15), (5,19)

4. a) A: 0, 4, 8, 12, 16  
 B: 17, 11, 9, 4, 1  
 C: 2, 3, 4, 3, 2  
 D: 4, 7, 4, 7, 4, 7

- b) i) C  
 ii) A  
 iii) D  
 iv) B

5. a) A:  
 (1,10), (2,9), (3,6), (4,4), (5,4), (6,3)  
 10, 9, 6, 4, 4, 3  
 B:  
 (1,10), (2,9), (3,8), (4,7), (5,6), (6,5)  
 10, 9, 8, 7, 6, 5  
 C:  
 (1,2), (2,1), (3,6), (4,5), (5,4), (6,3)  
 2, 1, 6, 5, 4, 3

- D:  
 (1,5), (2,6), (3,7), (4,8), (5,9), (6,10)  
 5, 6, 7, 8, 9, 10

- b) B

## AP Book PA7-20

page 85

1. a) Teacher to check.  
 b) i) No  
 ii) Yes  
 iii) Yes  
 iv) No  
 v) Yes  
 vi) No

2. Teacher to check.  
 3. Teacher to check.

4. Q2: lines go up from left to right  
 Q3: lines go down from left to right

## INVESTIGATION 1

- A. ii), iii), and v) are linear  
 Gaps ii) +3  
 iii) +3  
 v) -2

- B. i), iv), and vi) aren't linear  
 Gaps i) +3, +2, +6, +3  
 iv) +1, +1, +2, +4  
 vi) -4, -2, -4, -1

- C. The sequence is linear if and only if all of the gaps are equal.

5. a) Sequence B is linear because the gaps are all -3.

- b) Teacher to check.

6. a) i) +2, +2  
 ii) -3, -3  
 iii) +2, +5

- b) i) and iii)  
 c) i) and ii)  
 d) A → ii)  
 B → iii)  
 C → i)

## INVESTIGATION 2

- A. i) Sequence: 1, 3, 5, 7  
 Gaps: +2, +2, +2  
 ii) S: 4, 5, 6, 7  
 G: +1, +1, +1  
 iii) S: 1, 2, 4, 8  
 G: +1, +2, +4  
 iv) S: 2, 7, 4, 9  
 G: +5, -3, +5  
 v) S: 2, 7, 5, 2  
 G: +5, -2, -3  
 vi) S: 12, 10, 8, 6  
 G: -2, -2, -2  
 vii) S: 2, 6, 18, 54  
 G: +4, +12, +36  
 viii) S: 96, 48, 24, 12  
 G: -48, -24, -12

- B. i), ii) and vi)

- C. Teacher to check.

- D. The sequence is linear as long as the rule says to add or subtract a consistent number.

## AP Book PA7-21

page 88

1. a) Graph A

Input	Output
1	2
2	5
3	8
4	11

Graph B

Input	Output
1	3
2	4
3	5
4	6

Graph C

Input	Output
1	1
3	3
5	5
7	7

- b) A:  $O = (3 \times I) - 1$   
 B:  $O = I + 2$   
 C:  $O = I$

2. Teacher to check.

Sample answers:

- a) (3, 7)  
 b) (5, 5)  
 c) (4, 5)

3. Teacher to check.

4. a) 4¢  
 b) 2¢  
 c) 20¢  
 d) 3 minutes  
 e) 1¢

5. a) 20 km  
 b) 40 km  
 c) Yes, between hours 3 & 4 – the distance doesn't change so Kathy isn't moving.  
 d) No: 10 km/h for the first 3 hrs, 5 km/h for the last 2 hrs

6. a) 40 m  
 b) 60 m

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- c) Tom won, by 40 m.  
 d) 40 m  
 e) 15 seconds
7. a) i) \$8  
 ii) \$12  
 iii) \$10  
 b) \$4  
 c) Mike's (\$10 versus \$10.50 at Vi's)

## AP Book PA7-22

page 90

1.  $n = 2 \rightarrow 5$   
 $n = 3 \rightarrow 8$   
 $n = 4 \rightarrow 11$

2. a)

n	2n + 3
1	5
2	7
3	9
4	11
5	13

b)

n	3n - 2
1	1
2	4
3	7
4	10
5	13

c)

n	2n - 1
1	1
2	3
3	5
4	7
5	9

3. Teacher to check graphs.

a)

n	2n + 2
1	4
2	6
3	8
4	10
5	12

(1,4), (2,6), (3,8),  
 (4,10), (5,12)

b)

n	3n - 1
1	2
2	5
3	8
4	11
5	14

(1,2), (2,5), (3,8),  
 (4,11), (5,14)

c)

n	4n - 3
1	1
2	5
3	9
4	13
5	17

(1,1), (2,5), (3,9),  
 (4,13), (5,17)

## AP Book PA7-23

page 91

1. b) S: 1, 4, 7, 10, 13  
 G: +3  
 R: Start at 1, then add 3 each time.
- c) S: 21, 17, 13, 9, 5  
 G: -4  
 R: Start at 21, then subtract 4 each time.
- d) S: 2, 7, 12, 17, 22  
 G: +5  
 R: Start at 2, then add 5 each time.
- e) S: 16, 13, 10, 7, 4  
 G: -3  
 R: Start at 16, then subtract 3 each time.

2. Only i) can be found without calculation.  
*Sample explanation:*  
 If the term =  $a + (b \times \text{term number})$ , then  $b$  is always the gap in sequence.
3. If the rule is: "Start at  $a$ , then add/subtract  $b$  each time," then...
- a)  $b$  is the gap  
 b)  $a$  is the first term

c) we can easily continue the rule until the 10<sup>th</sup> term

4. If the formula is: term =  $a + (b \times \text{term number})$ , then...
- a)  $b$  is the gap  
 b)  $a + b$  is the 1<sup>st</sup> term  
 c)  $a + 10b$  is the 10<sup>th</sup> term
5. Answers may vary but the formula is generally better since it's quicker and there's less room for error.

## AP Book PA7-24

page 92

1. b)

FN	# Blocks
1	2
2	4
3	6

Rule:  
 Multiply the Figure Number by 2.

c)

FN	# Blocks
1	3
2	6
3	9

Rule:  
 Multiply the Figure Number by 3.

d)

FN	# Blocks
1	2
2	4
3	6

Rule:  
 Multiply the Figure Number by 2.

2. Circle: 1<sup>st</sup> and 3<sup>rd</sup> charts
3. b)  $4 \times \text{FN}$   
 $4 \times \text{FN} + 2$   
 c)  $2 \times \text{FN}$   
 $2 \times \text{FN} + 1$   
 d)  $3 \times \text{FN}$   
 $3 \times \text{FN} + 2$   
 e)  $3 \times \text{FN}$   
 $3 \times \text{FN} + 1$
4. Teacher to check.

## AP Book PA7-25

page 94

1. a) 7, 11, 15  
 Gap: +4  
 b) 1, 4, 7  
 Gap: +3  
 c) 6, 11, 16  
 Gap: +5  
 d) 6, 16, 26  
 Gap: +10  
 e) The gap is equal to the number that you multiply by in the rule or formula.

2. a)

FN	# Blocks
1	2 + 3 = 5
2	4 + 3 = 7
3	6 + 3 = 9

Gap: +2  
 Formula:  $2 \times \text{FN} + 3$

b)

FN	# Blocks
1	2 + 1 = 3
2	4 + 1 = 5
3	6 + 1 = 7

Gap: +2  
 Formula:  $2 \times \text{FN} + 1$

c)

FN	# Blocks
1	3 + 1 = 4
2	6 + 1 = 7
3	9 + 1 = 10

Gap: +3  
 Formula:  $3 \times \text{FN} + 1$

3. a)

TN	n × gap	T
1	4	11
2	8	15
3	12	19

Add: 7 Gap: +4  
 Rule: Multiply by 4, then add 7.

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b)

TN	$n \times \text{gap}$	T
1	2	5
2	4	7
3	6	9

Add: 3 Gap: +2

Rule: Multiply by 2, then add 3.

c)

TN	$n \times \text{gap}$	T
1	5	4
2	10	9
3	15	14

Subtract: 1 Gap: +5

Rule: Multiply by 5, then subtract 1.

d)

TN	$n \times \text{gap}$	T
1	4	2
2	8	6
3	12	10

Subtract: 2 Gap: +4

Rule: Multiply by 4, then subtract 2.

4. b)

TN	$n \times \text{gap}$	T
1	3	1
2	6	4
3	9	7
4	12	10

Formula:  $3n - 2$

c)

TN	$n \times \text{gap}$	T
1	4	3
2	8	7
3	12	11
4	16	15

Formula:  $4n - 1$

d)

TN	$n \times \text{gap}$	T
1	10	4
2	20	14
3	30	24
4	40	34

Formula:  $10n - 6$

e)

TN	$n \times \text{gap}$	T
1	2	4
2	4	6
3	6	8
4	8	10

Formula:  $2n + 2$

f)

TN	$n \times \text{gap}$	T
1	6	9
2	12	15
3	18	21
4	24	27

Formula:  $6n + 3$

g)  $2n + 1$

h)  $11n - 2$

i)  $10n + 7$

j)  $3n + 9$

k)  $12n - 10$

l)  $2n + 51$

5. Formula:  $5n + 1$

So in the 30<sup>th</sup> figure, there are 151 toothpicks.

## AP Book PA7-26

page 97

1. b) +2

c) +3

d) -2

2. a) +3

b) -4

c) +2

d) -3

e) +5

3. a) 0, 4, 10, 12

Gaps: +4, +6, +2

b) Term values

c) Vertical axis

4. a) i) +4

ii) +2

iii) +3

b) i)  $4n + 2$

ii)  $2n + 3$

iii)  $3n + 4$

5. a) i) 7

ii) 5

iii) 2

b) i) 2

ii) 5

iii) 7

c) i) C

ii) B

iii) A

6. B, C, A

7. Teacher to check.

## AP Book PA7-27

page 99

1. a) From left to right: 7, 4

b) From left to right: 2, 9

2. a) 12

b) 32

3. a) 20

b) 13

4. a) 25

b) 33

5. a) 40n

b)

T (n)	D (km)
1	40
2	80
3	120
4	160
5	200

c) Teacher to check.

d) 140 km

e) 8 hours

6. a) Teacher to check.

b) i) 25

ii) 28

iii) 27

c) i)  $n = 8$

ii)  $n = 7$

iii)  $n = 8$

d) i)  $4(8) + 1 = 32 + 1 = 33$

ii)  $5(7) - 2 = 35 - 2 = 33$

iii)  $3(8) + 9 = 24 + 9 = 33$

## AP Book PA7-28

page 101

1. a)  $5n - 1, 49$

b)  $6n - 2, 58$

c)  $6n - 3, 57$

d)  $11n - 6, 104$

e)  $10n + 1, 101$

f)  $2n + 11, 31$

g)  $7n - 4, 66$

h)  $3n + 41, 71$

2. Rule:

# of letters in  $n$ th row =  $n + 1$

K is the 11<sup>th</sup> letter so there will be 12 K's.

3. b) i) 4, 9, 14

ii)  $5 \times FN - 1$

iii)  $5n - 1$

iv)  $5(15) - 1 = 74$

c) i) 6, 8, 10

ii)  $2 \times FN + 4$

iii)  $2n + 4$

iv)  $2(15) + 4 = 34$

d) i) 4, 8, 12

ii)  $4 \times FN$

iii)  $4n$

iv)  $4(15) = 60$

4. a)  $3n + 1$

b) i)  $3(23) + 1 = 69 + 1 = 70$

ii) 20<sup>th</sup> term:  $3(20) + 1 = 61$

Add 3 to extend the sequence:

21<sup>st</sup> term: 64

22<sup>nd</sup> term: 67

$\therefore$  23<sup>rd</sup> term: 70

5. a)

1	$2^2 - 1^2 = 3$
2	$3^2 - 2^2 = 5$
3	$4^2 - 3^2 = 7$
4	$5^2 - 4^2 = 9$
5	$6^2 - 5^2 = 11$
6	$7^2 - 6^2 = 13$

b)  $2n + 1$

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- c) It will be in the  $22^{\text{nd}}$  row since the  $n^{\text{th}}$  row contains the expression:  $(n + 1)^2 - n^2$
- d)  $2(22) + 1 = 45$
- e)  $23^2 - 22^2 = 529 - 484 = 45$
6. b) i) 13, 17, 21  
ii)  $4n + 9$   
iii)  $4(10) + 9 = 49$
- c) i) 5, 7, 9  
ii)  $2n + 3$   
iii)  $2(10) + 3 = 23$
7. a)  $3n + 1$   
 $3(20) + 1 = 61$
- b)  $3n + 3$   
 $3(20) + 3 = 63$
- c)  $5n + 1$   
 $5(20) + 1 = 101$
- d)  $5n - 1$   
 $5(20) - 1 = 99$

**BONUS**

- i)  $2n + 10$
- ii)  $2n + 10 = 26$   
 $2n = 16$   
 $n = 8$
- So the  $8^{\text{th}}$  figure has a perimeter of 26.

8. *Formula:*  $5n$   
So in Figure 8, there will be  $8(5) = 40$  dots.

9. a)  $\frac{n}{n + 1}$
- b)  $\frac{2n + 3}{4n + 3}$

**BONUS**  $\frac{3n}{3n + 2}$

10.

Shaded	Unshaded
$4n + 4$	$n^2$
$4(7) + 4 = 32$	$(7)^2 = 49$

11. Tank 1 drains at a rate of 40L/min.  
After  $n$  minutes, the water left in Tank 1 is  $540 - 40n$ .  
It will empty after 13.5 minutes (solve by setting  $540 - 40n = 0$ ).  
Tank 2 doesn't drain at a consistent rate so we must extend the sequence to see when it will empty:  
500, 490, 470, 440, 400, 350, 290, 220, 140, 50, -50, ...  
It will empty after about 9.5 minutes.  
 $\therefore$  Tank 2 will empty first.

12. a) at 5 tables  $\rightarrow$  20  
at 6 tables  $\rightarrow$  22
- b) The # of people increases each time but the gap varies depending on the number of tables.  
When the number of tables goes from odd to even, the gap is +2.  
When the number of tables goes from even to odd, the gap is +4.  
So the pattern of the gap is +2, +4, +2, +4, ...
- c) at 8 tables  $\rightarrow$   
 $3(8) + 4 = 28$  people

**BONUS**

*Formula:*  $6n + 4$   
for 20 tables,  $n = 10 \rightarrow$   
 $6(10) + 4 = 64$  people

13. a) i)  $5^{\text{th}}$  triangle: 15  
 $6^{\text{th}}$  triangle: 21
- ii) the gap follows this sequence: +2, +3, +4, ...  
*Rule:*  
Start at 2, add 1 each time.
- iii)  $7^{\text{th}}$  triangle: 28  
 $8^{\text{th}}$  triangle: 36

- b) i)  $5^{\text{th}}$  triangle: 25  
 $6^{\text{th}}$  triangle: 36
- ii) the gap follows this sequence: +1, +3, +5, ...  
*Rule:*  
Start at 1, add 2 each time.
- iii)  $7^{\text{th}}$  triangle: 49  
 $8^{\text{th}}$  triangle: 64

14. a)  $3n + 4$   
b)  $5n - 4$   
c)  $7n - 4$   
d)  $2n + 9$
15. a)  $4n - 2$   
 $4(50) - 2 = 198$
- b)  $8n + 2$   
 $8(50) + 2 = 402$
16. a) Teacher to check.
- b)  $n(n+1) = 15(16) = 240$
- c) This will be equal to half the sum of the first 15 even numbers:  
 $240 \div 2 = 120$

17. a)

FN	S	U
1	1	0
2	3	1
3	6	3
4	10	6

- b) Continue sequence for shaded triangles: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55  
 $\therefore$  the  $10^{\text{th}}$  figure has 55 shaded triangles
18. a) The graph could only represent the first situation.  
The cost of renting a bike for  $n$  hours increases linearly (that is, the gap is always the same).  
The area of a square with side length  $n$  doesn't increase linearly.

- b) Teacher to check.